Frequency Stabilization of a Microwave Oscillator with an External Cavity*

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Summary—**This paper describes a procedure by which a cavity stabilizer may be designed for a microwave oscillator. Formulas are derived for the following essential design parameters: 1) stabilization factor; 2) stabilization range; 3) vswr of the stabilizer circuit with cavity tuned; 4) vswr of the stabilizer circuit with the cavity detuned; and 5) insertion loss of the cavity assembly.**

The validity of designing with the derived relations has been experimentally confirmed.

Introduction

IN MANY applications, the need arises for a microwave oscillator having a stable frequency characteristic both on a long and short-time basis. The addition of an external cavity and associated microwave circuitry to an ordinary magnetron or klystron transforms these tubes into frequency stabilized oscillators. The general scheme has been investigated by many.[1] Although this paper concerns itself with a stabilizer configuration which has been previously studied,[2] a more than cursory survey of the literature does not show a precise analysis and quantitative development such as is here described.[3] The validity of the results justifies their collation. The configuration in Fig. 1 is one in which the external cavity and damping resistor[4] are either in series or parallel with the oscillator output line. The damping resistor consists of a matched load and an inductive or capacitive susceptance placed so that there is a conductance in series with the stabilizing cavity. The purpose of the damping resistor is to suppress unwanted modes in the tube, the transmission line, and stabilizing cavity assembly. The resistor also increases the stabilization range.

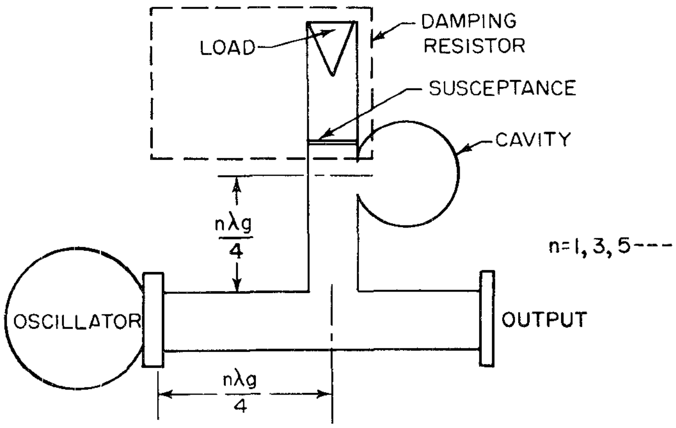

Fig. 1—Cavity stabilized oscillator.

List of Symbols

- S = stabilization factor.
- m = junction transformer ratio.
- G_T = damping resistor (normalized).
- Q_{EC} = external Q of the cavity.
- Q_{OC} = unloaded Q of the cavity.
- Q_{EM} = external Q of the oscillator.
- ρ_{off} = vswr into assembly with cavity detuned.
- ρ_{on} = vswr into assembly with cavity tuned.
- L = insertion loss of the stabilizing system.
- f_0 = system frequency.
- ΔF = stabilization frequency range.
- PF = pulling figure for the oscillator.

The design relations are given below (their derivation will be presented in the Appendix):

$$S = m^2\left[\frac{(G_T)^2}{\left(G_T + \frac{Q_{EC}}{Q_{OC}}\right)^2}\,\frac{Q_{EC}}{Q_{EM}}\right] + 1 \tag{1a}$$

$$L = 20\log\left[1 + \frac{m^2 G_T \frac{Q_{EC}}{Q_{OC}}}{2\left(G_T + \frac{Q_{EC}}{Q_{OC}}\right)}\right] \tag{1b}$$

$$\rho_{\text{on}} = \left[1 + \frac{m^2 G_T \frac{Q_{EC}}{Q_{OC}}}{G_T + \frac{Q_{OC}}{Q_{OC}}}\right] \tag{1c}$$

$$\rho_{\text{off}} = m^2 G_T + 1 \tag{1d}$$

$$\text{PF} = \frac{0.417 f_0}{Q_{EM}} \tag{1e}$$

$$\Delta F = f_0\left(\frac{G_T}{Q_{EC}} + \frac{1}{Q_{OC}}\right). \tag{1f}$$

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[1] F. F. Rieke, "Microwave Magnetrons," M.I.T. Rad. Lab. Ser. No. 6, McGraw-Hill Book Co., Inc., New York, N. Y., 1948. See Ch. 16.
W. V. Smith, "Magnetron Tuning and Stabilization," M.I.T. Rad. Lab. Rep. No. 567, July 13, 1944.
W. M. Preston and J. B. Platt, "Magnetron Stabilizer Tuner," Rad. Lab. Group Rep. No. 473, 1943.
W. M. Preston, "Magnetron Stabilizing Tuner," Rad. Lab. Group Rep. No. 71, May 31, 1944.
E. Shelton, "Stabilization of microwave oscillators," IRE Trans., vol. ED-1, pp. 30–40; December, 1954.

[2] Preston and Platt, *op. cit.*

[3] The results developed have been independently derived by E. Shelton of Raytheon in a private communication.

[4] Rieke, *op. cit.*

Before the actual design of the stabilizer circuit can be undertaken, there are some fundamental requirements of the oscillator and system to be considered.

For the oscillator, the starting characteristic is important. Upon start of oscillation, the tube is exposed to the mismatch of the stabilizer cavity detuned from resonance. The maximum value of this vswr ρ_{off}, must be known for the tube. In the case of pulse tubes electrical breakdown in the cavity is an important consideration. The resonator dimensions must be such that the voltage gradient in the cavity is safely below the breakdown level.

For the system, the following listed properties are important:

1) Stabilization factor required;
2) The insertion loss that can be tolerated in the stabilized circuit;
3) Stabilization range:
   a) This range must be large enough to prevent unlocking the stabilized tube by external match variations at the tube output;
   b) The range must also be sufficiently large to allow for an automatic frequency control loop either from the cavity to tune in the oscillator or from the oscillator to control the cavity.

The procedure for determining all the necessary quantities in design relations (1) is given in the following paragraphs.

1) The transformer ratio m of the junction can be determined from a vswr measurement of the oscillator input to the junction by itself with matched loads on the other two arms.

2) Q_{EC} the cavity external Q, and Q_{OC}, the cavity unloaded Q, can be determined by standard methods. The cavity must be of the line terminating absorption type.

3) Q_{EM}, the oscillator external Q, can be determined from a pulling figure measurement. The pulling figure is defined as the maximum excursion in frequency for a vswr of 1.5 moved through a wavelength. From (1c), Q_{EM} can be computed.

4) For a value of ρ_{off} (known for the oscillator) and a measured junction transformer ratio m the value of damping resistor G_T is determined from (1d).

$$G_T = \frac{\rho_{off} - 1}{m^2}.$$

The location and value of the susceptance necessary to yield the computed value of G_T at the cavity is then determined in the manner illustrated by the following example (see Fig. 2).

For $\rho_{off}=3$ and $m^2=0.7$ using (1d) we obtain $G_T=2.86$. The remaining problem is to determine the value and position of the admittance that will yield the computed value of G_T at the cavity. Using the Smith

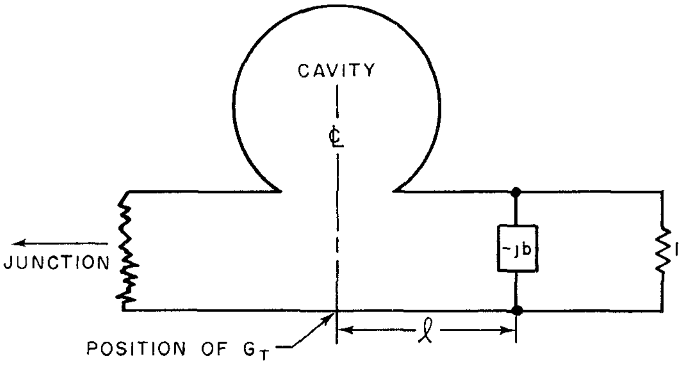

Fig. 2—Circuit for computing susceptance.

chart in Fig. 3 (opposite) and moving from $G_T=2.86$ toward the load to $1-jb$ point yields the value and position of inductive susceptance necessary; namely, $-j\,1.1$ at a distance of 0.416 λg from the center of the cavity input.

Now with all the quantities known, one can determine the necessary characteristics of the stabilized oscillator. The calculated values are compared to the experimental results on Figs. 4–6 (p. 60). The data was obtained on a cw magnetron.

Appendix

The analysis to follow is based on the equivalent circuit of oscillator and cavity combination shown in Fig. 7.

Expressing the admittance of a tuned circuit as $Y=G(1+jQ_0\epsilon)$ and also defining the other relations as shown below.

$$Q_E = p^2 y \qquad y = \sqrt{\frac{C}{L}}$$

$$Q_O = Ry \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\epsilon = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \approx \frac{2\Delta\omega}{\omega_0}$$

$$Y_{(4-4)} = p^2(Y_C) \tag{2}$$

where

$$Y_C = G_C(1 + jQ_{0C}\epsilon)$$

$$Y_{(4-4)} = p^2 G_C(1 + jQ_{0C}\epsilon) \tag{3}$$

$$Y_{(4-4)} = \frac{Q_{EC}}{Q_{OC}}(1 + jQ_{0C}\epsilon) \tag{4}$$

$$Z_{(4-4)} = \frac{1}{\frac{Q_{EC}}{Q_{OC}}[1 + jQ_{0C}\epsilon]} \tag{5}$$

$$Z_{(4-5)} = \frac{1}{\frac{Q_{EC}}{Q_{OC}}[1 + jQ_{0C}\epsilon]} + \frac{1}{G_T}. \tag{6}$$

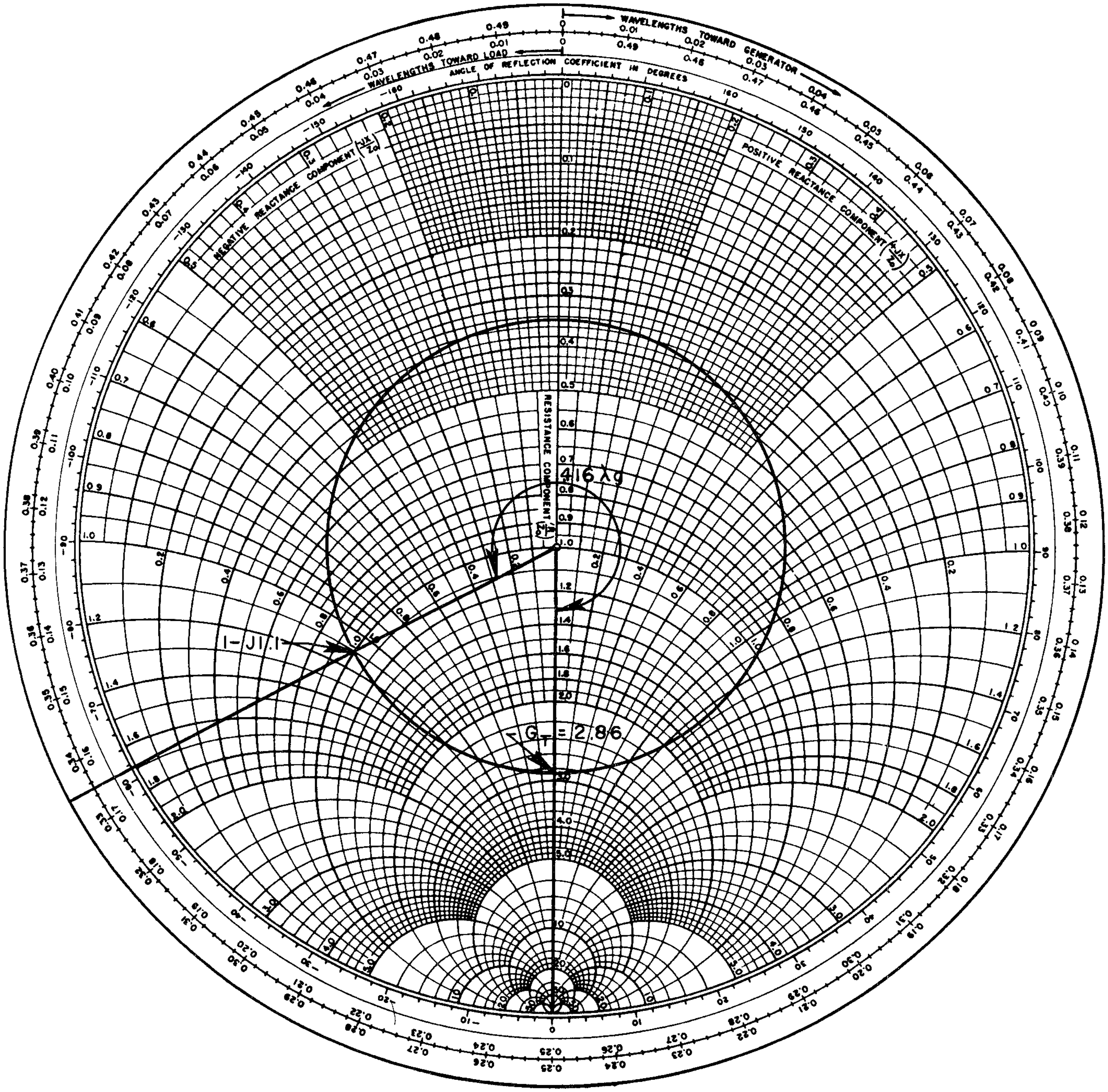

Fig. 3—Determination of the susceptance to yield the correct value of damping resistor.

Let

$$V = \frac{Q_{EC}}{Q_{OC}}$$

and recombining the terms of (6)

$$Z_{(4-5)} = \frac{G_T + (V + jQ_{EC}\epsilon)}{(V + jQ_{EC}\epsilon)G_T} \tag{7}$$

but because of

$$\frac{n\lambda g}{4} \qquad n = 1, 3, 5 \cdots$$

$$Z_{(2-2)} = \frac{m^2}{Z_{(4-5)}} = \frac{m^2 G_T(V + jQ_{EC}\epsilon)}{G_T + (V + jQ_{EC}\epsilon)} \tag{8}$$

$$Z_{(2-3)} = \frac{m^2(V + jQ_{EC}\epsilon)G_T}{G_T + (V + jQ_{EC}\epsilon)} + 1 \tag{9}$$

$$Z_{(1-1)} = \frac{1}{Z_{(2-3)}} \qquad Y_{(1-1)} = Z_{(2-3)}$$

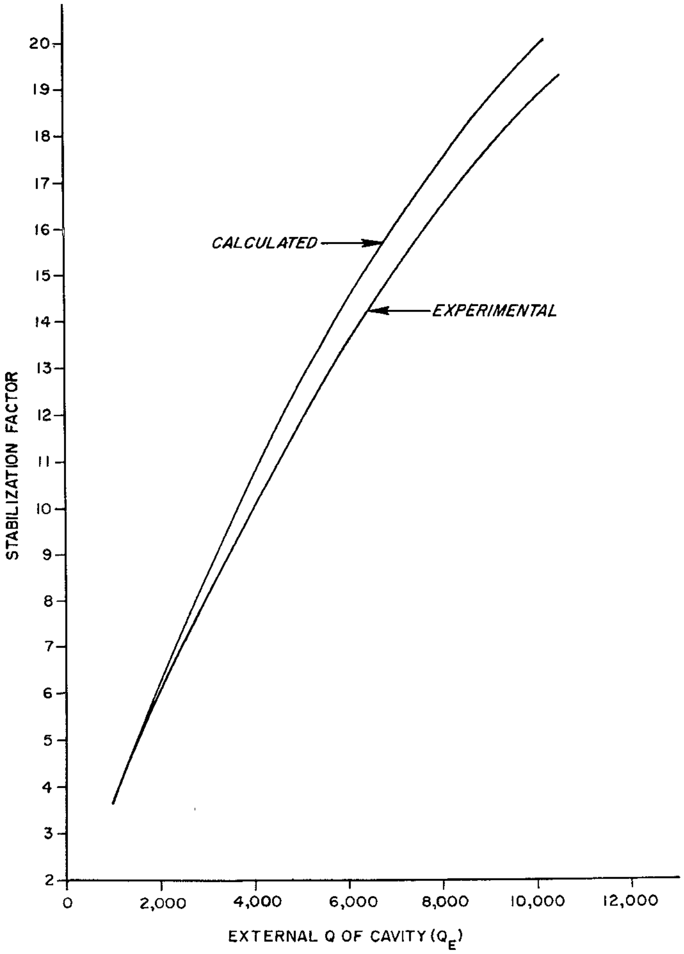

Fig. 4—Stabilization factor characteristic.

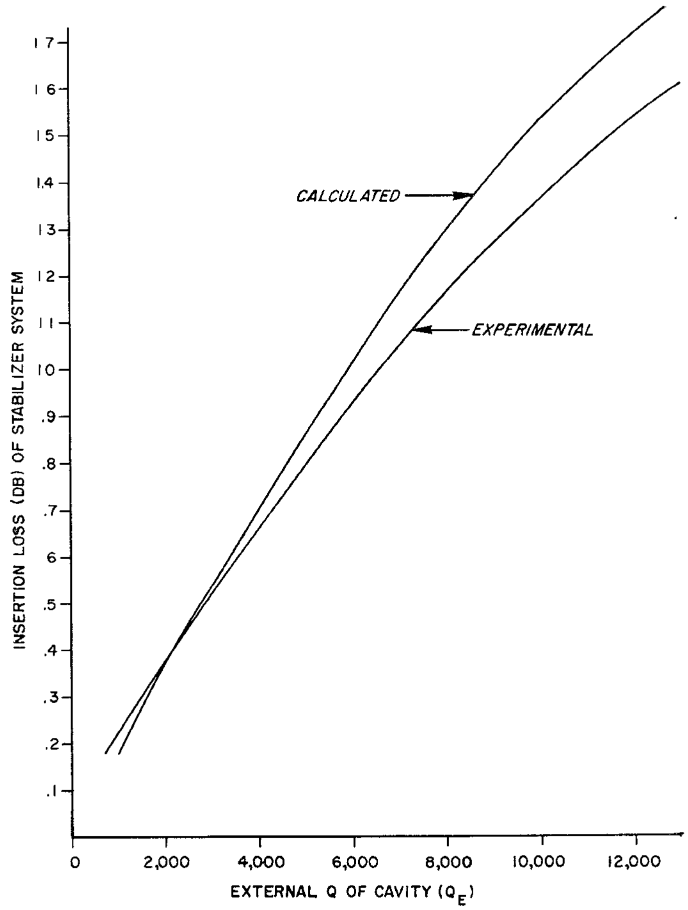

Fig. 5—Insertion loss characteristic.

again because

$$\frac{n\lambda g}{4} \qquad n = 1, 3, 5 \cdots$$

$$Y_{(1-1)} = Z_{(2-3)} = \frac{(m^2VG_T+G_T+jm^2Q_{EC}\epsilon G_T+jQ_{EC}\epsilon)(G_T+jVQ_{EC}\epsilon)}{(G_T+V)^2+(Q_{EC}\epsilon)^2}. \quad (10)$$

Finding the imaginary part of $Y_{(1-1)}$ yields

$$B_{(1-1)} = \frac{m^2Q_{EC}\left(\frac{2\Delta\omega}{\omega_0}\right)G_T^2}{(G_T+V)^2+\left(Q_{EC}\frac{2\Delta\omega}{\omega_0}\right)^2}. \quad (11)$$

Differentiating (11) with respect to $\Delta\omega$ yields

$$\frac{dB_{(1-1)}}{d\Delta\omega} = \frac{\left[m^2(G_T+V)^2+4Q_{EC}^2\left(\frac{\Delta\omega}{\omega_0}\right)^2\right]\left[\frac{2Q_{EC}^2G_T^2}{\omega_0}\right]}{\left[(G_T+V)^2+4Q_{EC}^2\left(\frac{\Delta\omega}{\omega_0}\right)^2\right]^2}. \quad (12)$$

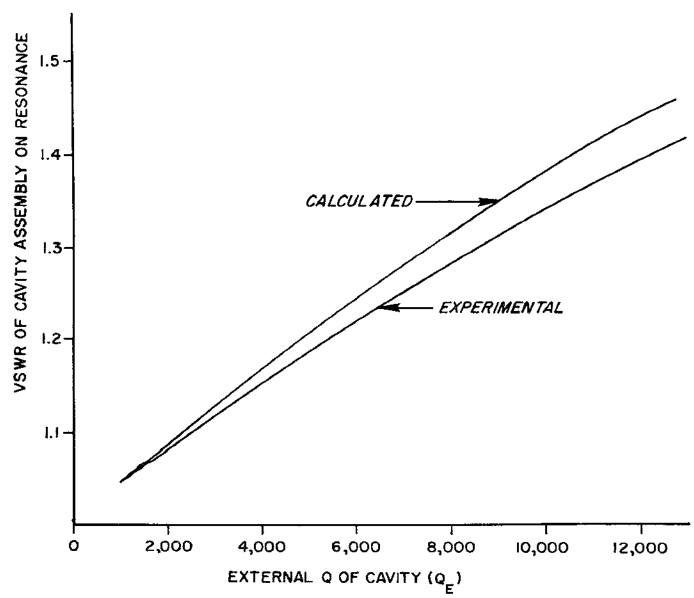

Fig. 6—Input vswr characteristic.

Let $\Delta\omega \to 0$

$$\frac{dB_{(1-1)}}{d\Delta\omega} = \frac{2m^2Q_{EC}(G_T)^2}{\left(G_T+\frac{Q_{EC}}{Q_{OC}}\right)^2\omega_0}. \quad (13)$$

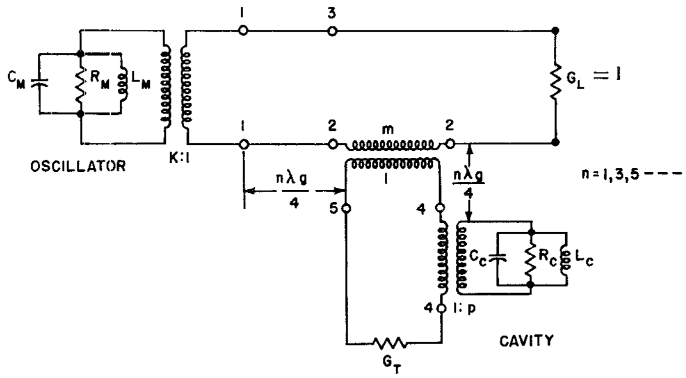

Fig. 7—Oscillator and cavity equivalent circuit.

Consideration of the oscillator circuit yields

$$Y_{(1-1)M} = K^2 G_M[1 + jQ_{OM}\epsilon]. \tag{14}$$

In a manner similar to operation of (4)

$$Y_{(1-1)M} = \frac{Q_{EM}}{Q_{OM}}[1 + jQ_{OM}\epsilon] \tag{15}$$

$$B_{(1-1)M} = Q_E\epsilon = 2Q_{EM}\frac{\Delta\omega}{\omega_0} \tag{16}$$

$$\frac{dB_{(1-1)M}}{d\Delta\omega} = \frac{2Q_{EM}}{\omega_0}. \tag{17}$$

The stabilization factor is defined as

$$S = \frac{\dfrac{dB_{(1-1)}}{d\Delta\omega}}{\dfrac{dB_{(1-1)M}}{d\Delta\omega}} + 1. \tag{18}$$

Substituting (13) and (17) into (18) yields the relation for stabilization factor.

$$S = m^2\frac{Q_{EC}}{Q_{EM}}\left[\frac{G_T}{G_T + \dfrac{Q_{EC}}{Q_{EC}}}\right]^2 + 1. \tag{19}$$

The stabilization range is defined as the frequency spread between the peaks of the transformed cavity susceptance at the oscillator terminals. Fig. 8 illustrates the curve. Stabilization depends upon the transformed cavity and magnetron susceptance having the same slope.

Since at the peaks of the transformed susceptance curve

$$\frac{dB_{(1-1)}}{d\Delta\omega} = 0, \tag{20}$$

we can obtain an expression for the frequency spread by setting (12) equal to zero and solving for ΔF.

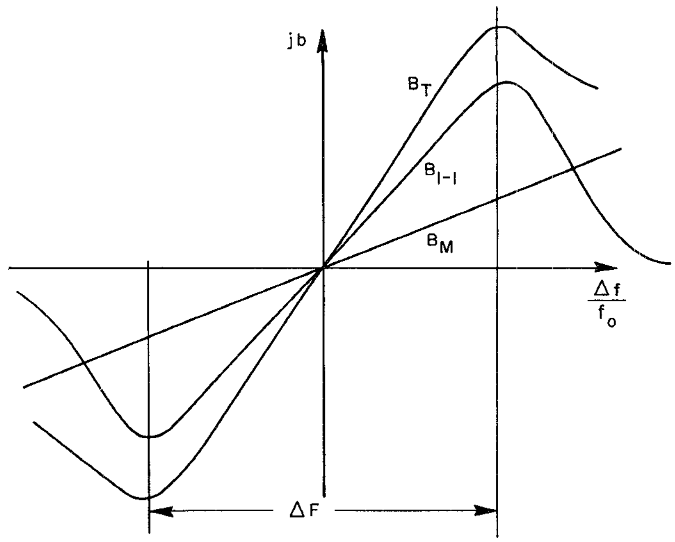

Fig. 8—Cavity and magnetron susceptance characteristics.

The solution yields the total stabilization range namely

$$\Delta F = f_0\left[\frac{G_T}{Q_{EC}} + \frac{1}{Q_{OC}}\right]. \tag{21}$$

The off resonance vswr looking in the stabilizing cavity assembly is derived from the following:

Using (9) off resonance $\epsilon \rightarrow \infty$

$$Z_{(2-3)} = \frac{\left[\dfrac{m^2V}{\epsilon} + jQ_{EC}G_T\right]}{\left[\dfrac{G_T}{\epsilon} + \dfrac{V}{\epsilon} + jQ_{EC}\right]} + 1 \tag{22}$$

$$Z_{(2-3)} = m^2G_T + 1 \tag{23}$$

$$\rho_{\text{off}} = m^2G_T + 1 \tag{24}$$

is the expression for the vswr into the assembly with the cavity detuned. The relation for the expected vswr with the cavity tuned is derived also using (8).

Using (8) and letting $\Delta\omega \rightarrow 0$ yields

$$Z_{(2-2)} = \frac{m^2G_T\dfrac{Q_{EC}}{Q_{OC}}}{G_T = \dfrac{Q_{EC}}{Q_{OC}}} \tag{25}$$

$$\rho_{\text{on}} = Z_{(2-2)} + 1 \tag{26}$$

$$\rho_{\text{on}} = \frac{m^2G_T\dfrac{Q_{EC}}{Q_{OC}}}{\left(G_T + \dfrac{Q_{EC}}{Q_{OC}}\right)} + 1. \tag{27}$$

The loss in the stabilizer is calculated again with aid of Fig. 8. Using (25), the loss is determined using the

A, B, C, D, matrix[5]

$$U = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_{(2-2)} \\ 0 & 1 \end{bmatrix}. \quad (28)$$

The loss is given by

$$|R|^2 = \frac{(A+B+C+D)^2}{4}. \quad (29)$$

Substituting (25) into (29) yields

$$|R|^2 = \left[1 + \frac{m^2 G_T \dfrac{Q_{EC}}{Q_{OC}}}{2\left(G_T + \dfrac{Q_{EC}}{Q_{OC}}\right)}\right]^2 \quad (30)$$

but $L = 20 \log R$.

Substituting (30) in (31)

$$L = 20 \log \left[1 + \frac{m^2 G_T \dfrac{Q_{EC}}{Q_{OC}}}{2\left(G_T + \dfrac{Q_{EC}}{Q_{OC}}\right)}\right]. \quad (31)$$

[5] E. A. Guillemin, "Communication Networks," John Wiley and Sons, Inc., New York, N. Y., vol. 2, ch. 4, pp. 132–180; 1947.

Cooling of Microwave Crystal Mixers and Antennas*

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Summary—**The development of low-noise mixer crystals has reached the point where the noise figure is approaching fundamental, theoretical limits. The desire for still greater sensitivity has led to the consideration of other possible means for noise reduction. This paper will discuss two possibilities: physically cooling the mixer crystal, and using an antenna directed toward background noise which is lower than room temperature. The improvement which can be realized increases rapidly as the room-temperature noise figure is reduced.**

Receiver Sensitivity Improvement from "Cold Antenna"

The improvement ratio of receiver sensitivity as the antenna looks at free space, to the sensitivity as it looks at a room temperature background, increases hyperbolically as the noise figure is improved.

$$\frac{(N_0)T_0}{(N_0)T_s} = \frac{F}{F + \dfrac{T_s}{T_0} - 1} = [\mathrm{SR}]_A. \quad (1)$$

Here $(N_0)T_0$ is the output noise in the receiver as it looks at a room-temperature source, T_0, and $(N_0)T_s$ is the output noise in the receiver as it looks at the free-space background T_s. F represents the receiver noise figure. $[\mathrm{SR}]_A$ is the receiver sensitivity improvement ratio due to the cold antenna.

Because of the rapidity with which this ratio improves as F approaches its theoretical limit 1, it becomes economical to think of cooling the mixer *if* it has a very low F to begin with. X-band receivers using narrow-band techniques and low-noise germanium mixer crystals have noise figures low enough to make this improvement in sensitivity important and feasible.

Receiver Sensitivity Improvement from Cooling Mixer

The noise figure of a receiver using a crystal is given by

$$F = L_x[t_x + F_{if} - 1] \quad (2)$$

where F is the receiver noise figure, L_x is crystal conversion loss, t_x is crystal noise temperature, and F_{if} is the IF noise figure. This formula is not convenient for discussing improvements due to cooling because t_x is a function of L_x.[1] The functional relationship is

$$t_x = \begin{cases} \bar{t}\left(1 + \dfrac{1}{L_x}\right) + \dfrac{1}{L_x} & \text{narrowband} \\ \bar{t}\left(1 - \dfrac{2}{L_x}\right) + \dfrac{2}{L_x} & \text{broadband.} \end{cases} \quad (3)$$

Here $\bar{t}$ is the noise temperature due only to the crystal. Substituting back in (2)

$$F = [L_x\bar{t} - (1, 2)]\bar{t} + (1, 2) + L_x(F_{if} - 1) \quad (4)$$

where the 1 and 2 pertain to narrowband and broadband, respectively. Now

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[1] The harmonics of the local oscillator are assumed to be all shorted.